

---

LONG RANGE PLAN FOR MATHEMATICAL AND STATISTICAL SCIENCES  
FIELDS INSTITUTE REPORT

CONTENTS

1. Operational structure, funding and leveraging	2
2. Recent examples of major research advances through Fields Institute activities	2
3. Organisation of scientific events	4
4. Postdoctoral fellows	4
5. Graduate training	5
6. Prizes and Special Lectureships	5
7. Seminar/Colloquium Series and Working Groups	5
8. Centre for Mathematical Medicine (CMM)	6
9. Multi-disciplinary activities	6
10. Education	6
11. Targeting non-academic sectors and industrial collaboration	7
12. Public events	7
13. New initiatives	8

## 1. OPERATIONAL STRUCTURE, FUNDING AND LEVERAGING

The Institute's goal is to create opportunities for discovery, stimulate communication and foster cooperation in all areas of the mathematical sciences and at all levels of scientific activity, spanning research, education and industry. The Institute brings together students, postdoctoral fellows, junior and established researchers with the world's leading scientists in a given field. It provides a unique stimulating atmosphere for dissemination of ideas and intense collaboration in a building designed for mathematical communication.

The primary activities of the Institute are its Thematic Programs. These create unique opportunities of mathematical communication and collaboration for an extensive period of time. Fields currently organizes six month Thematic Programs and two month Summer Programs. Focus and Thematic Programs of one to two months in length are being planned for the near future. In addition, the Institute supports a wide range of programs of shorter duration such as workshops and conferences, short courses, summer schools, recurring seminar series, and special lectures. Such activities are sometimes held off-site, many of them on the campuses of our partner universities. All these involve participants from Canada and around the world, and include graduate students and postdoctoral fellows, as well as more senior and well-established scientists.

This is achieved through extremely effective leveraging on different levels – concerning sources of funding, as well as the involvement of the community in scientific activity. Regarding the financial aspect, NSERC support, which accounts for 25% of the Fields Institute budget, is leveraged through sources that would otherwise not be available at the current level, such as funding from the Ontario Ministry of Training, Colleges and Universities, and from 23 partner Universities associated with the Institute (cf. *infra*). A further particular example is the *Dean's Distinguished Visiting Lecturer Graduate Course*, a joint program with the University of Toronto; cf. 5 (3). The scientific activity generated as a result exceeds NSERC's contribution by several orders of magnitude. Indeed, the Institute creates in its Principal Sponsoring and Affiliate Universities a climate which engages faculty in scientific activity, stimulates research and generates new national and international collaboration. Moreover, the Institute plays a significant role in attracting star researchers as faculty to Universities in the region, thus helping to recruit scientists of the highest international calibre.

The Fields Institute's principal sources of funding (2010-2011) are the following:

- (1) Ontario Ministry of Training, Colleges and Universities (MTCU): \$2.0M
- (2) NSERC: \$1.2M
- (3) Sponsoring revenue from 7 Principal Sponsoring Universities and 16 Affiliate Universities: approximately \$560,000
- (4) Other grants (e.g., NSF, commercial and industrial mathematics grants): approximately \$420,000
- (5) MITACS: approximately \$65,000

## 2. RECENT EXAMPLES OF MAJOR RESEARCH ADVANCES THROUGH FIELDS INSTITUTE ACTIVITIES

The following highlights some of the recent spectacular research advances made through scientific activities at the Institute.

- The Institute's Thematic Programs have had a transformative impact on the development of entire mathematical fields, and on establishing links between existing disciplines. Prime examples are the year-long *1996-97 Thematic Program on Algebraic Model Theory* and the *Winter 1997 Thematic Program on Singularity Theory and Geometry*. The fruitful interaction between both areas which flourished during these (partially) concurrent Programs contributed tremendously to the development of  $\mathfrak{o}$ -minimality and nurtured its interplay with real analytic geometry: methods from the latter field have been adapted to the  $\mathfrak{o}$ -minimal setting, and, conversely, model-theoretic techniques stemming from the  $\mathfrak{o}$ -minimal point of view have led to new discoveries in real analytic geometry. The mutual stimulation of these areas culminated recently in the highly successful *Winter 2009 Thematic Program on  $\mathfrak{o}$ -minimal Structures and Real Analytic Geometry*.

- The area of partial differential equations (PDEs) has enormous breadth, from its connections to applied mathematics and modelling to its impact on geometry and theoretical physics. During the (year long) *2003–04 Program on Partial Differential Equations* we succeeded in highlighting many of the major themes of current research in this area. More than 18 postdoctoral fellows spent the year at Fields, and they have gone on to careers in mathematics around the world, including in Canada and the United States of course, but also France, Sweden, China, Brazil, Nigeria and South Africa. Furthermore, our thematic program year has had a lasting effect on the Canadian mathematics community. It placed the country on the map of international centers of concentration on the mathematics of PDEs, enabling us to attract new faculty to our universities (UBC, University of Toronto, McGill, McMaster, Waterloo, Saskatoon, Victoria, and many others), and promoting our graduate programs in this specialization.

We shall highlight three of the major breakthroughs:

- (1) A. Biryuk, W. Craig and S. Ibrahim discovered a new estimate on the Fourier coefficients of weak solutions to the Navier–Stokes equations. This is a technical advance; however, it is significant because of the importance of the Navier–Stokes equations in mathematical modeling of fluids in many engineering applications, and as well in theories of turbulence. This estimate has had important subsequent implications, including upper bounds on the Kolmogorov spectrum for Navier–Stokes turbulence, upper and lower bounds on the inertial range (which is important in theories of phase space energy transport), and lower bounds on the dimension in phase space of Navier–Stokes singularities. This latter topic is relevant to the Millenium Prize Problem on singularities for the Navier–Stokes equations in three dimensions.
  - (2) A second important breakthrough is the quantification of solitary wave interactions for free surface water waves. This is work that started in a collaboration between P. Guyenne (Program postdoctoral fellow), J. Hammack, D. Henderson and C. Sulem. Unlike such idealized models such as the Korteweg deVries equations, waves do not experience elastic interactions, however quantifying the residual to their interaction has been a challenge. The result is a precise numerical simulation, backed by both experimental studies and by mathematical theorems. It is an outcome of the participation of D. Henderson in the thematic program, and the activities around the workshop on dynamic free surface water waves.
  - (3) A third breakthrough is the establishment of the point of view which creates a connection between the fields of PDEs and of Hamiltonian dynamical systems. This has had an important and lasting effect on the Canadian mathematics community, and has also propagated to the international community. Activities in Canada that stem from this include the sequence of Young Mathematicians’ Conferences (YMC) on PDEs and dynamical systems. Similar summer schools and conferences are now held worldwide, with events in Nanjing (China), Paris (France), and Naples (Italy), to mention some.
- More recent Thematic Programs have as well seen the solution of long-standing problems in the mathematical sciences.
    - During the *Fall 2007 Thematic Program on Operator Algebras*, U. Haagerup and M. Musat solved the Effros–Ruan Conjecture, formulated in 1991, establishing a Grothendieck-type inequality for bilinear forms on  $C^*$ -algebras. The result is of fundamental importance in the theory of operator algebras and operator spaces.
    - During the *Fall 2010 Thematic Program on Asymptotic Geometric Analysis*, V. Losert, M. Neufang, J. Pahl and J. Steprāns solved the Ghahramani–Lau Conjecture, formulated in 1994, showing that the measure algebra over any locally compact group is strongly Arens irregular. The proof relies on a blend of methods in harmonic analysis and Banach algebra theory.

### 3. ORGANISATION OF SCIENTIFIC EVENTS

Scientific events range from Thematic Programs lasting one, two or six months, to (mostly week-long) workshops, graduate courses, summer or winter schools and seminars.

3.0.1. *Thematic Programs.* The Institute's Thematic Programs comprise a series of workshops, a postdoctoral and visitor program, a series of linked graduate courses, mini-courses, young researchers and visitors seminars, and moreover often include a summer or winter school. The subjects are chosen for their extremely high current international interest and impact. Thematic Programs bring together most of the world's senior researchers and beginning scholars in a given area of the mathematical sciences for one to six months of intense communication and collaboration.

Thematic Programs are followed, two to three years later, by a Retrospective Workshop to analyze the impact of the Program on the field and its contribution to major scientific advances. One of the most exciting aspects of this analysis is that it reveals the career-shaping experiences graduate students and postdoctoral fellows gained through the Thematic Program: many of them attend the retrospective workshop at Fields now holding University positions.

The following list is to give an impression of the large spectrum of areas within the mathematical sciences covered through recent six and two month Fields Thematic Programs:

- (1) *Mathematics in Quantum Information* (July – August 2009)
- (2) *Foundations of Computational Mathematics* (July – December 2009)
- (3) *Quantitative Finance: Foundations and Applications* (January – June 2010)
- (4) *Mathematics of Drug Resistance in Infectious Diseases* (July – August 2010)
- (5) *Asymptotic Geometric Analysis* (July – December 2010)
- (6) *Dynamics and Transport in Disordered Systems* (January – June 2011)
- (7) *The Mathematics of Constraint Satisfaction* (July – August 2011)
- (8) *Discrete Geometry and Applications* (July – December 2011)

3.0.2. *Account for scientific activities.* As mentioned above, Retrospective Workshops, organized within two to three years of each Thematic Program, analyze the state-of-the-art of the given area and the impact the Program has had on its development. Recent examples include the following:

- (1) *Harmonic Analysis: A Retrospective Workshop*, May 31 – June 4, 2010
- (2) *Conference on Homotopy Theory and Derived Algebraic Geometry*, August 30 – September 3, 2010

The Institute further accounts for its scientific activities through its publications: the *Fields Institute Monographs* and *Fields Institute Communications* series, published jointly with the American Mathematical Society, as well as the Institute's newsletter *Fields Notes*, with three issues per year.

3.0.3. *Workshops.* Among the Institute's General Scientific Activities, workshops and conferences on a topic of great current interest play a major role. Typically of a duration of three to five days, these events offer the scientific community an extremely flexible framework for exchange of ideas on the pulse of research. 4965 scientists (Thematic Programs: 1077, General Scientific Activities: 2912, Mathematics Education Forum: 130, Commercial and Industrial Mathematics activities: 846) participated in Fields Institute workshops in 2010, with 2573 from abroad.

### 4. POSTDOCTORAL FELLOWS

Postdoctoral training is a key activity of the Institute. Postdoctoral fellows are attached to a Thematic Program and/or a group of researchers from the Institute's partner Universities. In 2010, the Institute sponsored 63 fellows funded either fully or partially. Examples of Fellowships:

- (1) Fields Postdoctoral Fellowships: Fields-Ontario Postdoctoral Fellowships, Marsden Postdoctoral Fellowships, Thematic Program Postdoctoral Fellowships, Fields Principal Sponsoring University Postdoctoral Fellowships
- (2) Fields Research Immersion Fellowships

## 5. GRADUATE TRAINING

An important feature of the Institute's activities has been a widening of the range of graduate and postdoctoral training. NSERC funding is largely used to leverage other sources, such as NSF support.

- (1) Thematic Program on Quantitative Finance: Foundations and Applications (January – June 2010)
  - (a) *Graduate Course on Numerical Methods in Finance*
  - (b) *Graduate Course on Interest rates and credit risk*
  - (c) *Graduate Course on Foundations of Mathematical Finance*
- (2) Thematic Program on Asymptotic Geometric Analysis (July – December 2010)
  - (a) *Graduate Course on Isoperimetric Inequalities and Applications to Asymptotic Geometric Analysis*
  - (b) *Graduate Mini-course: An Introduction to Infinity Harmonic Functions*
  - (c) *Mini-course on Optimal Transportation: Geometry, Regularity and Applications*
- (3) *Dean's Distinguished Visiting Lecturer Graduate Course*: This annual Program is funded jointly by Fields and the University of Toronto's Faculty of Arts and Science and Department of Mathematics. It brings one of the world's leading scientists to the Institute to give a full semester graduate course.
- (4) *Commercial and Industrial Mathematics Courses*
  - (a) *Quantitative Wealth Management Analytics Group Inc. (QWeMA) Short Course on Retirement Income Analytics*, January 7, 2010
  - (b) *Affine Schubert Calculus Summer School*, July 7–10, 2010

## 6. PRIZES AND SPECIAL LECTURESHIPS

The Institute awards several Prizes and Special Lectureships recognizing exceptional contributions in the mathematical sciences. Its Special Lecture Series as well as the Dean's Distinguished Visitor Program bring the world's top mathematical scientists to Canada.

- (1) *CRM-Fields-PIMS Prize*
- (2) *Fields Institute Fellows*
- (3) *Fields Distinguished Lecture Series* (cf. 12 below)
- (4) *Coxeter Lecture Series*
- (5) *Fields Distinguished Lecture Series in Statistical Science*
- (6) *Dean's Distinguished Visitor Program* (cf. 5 below)

## 7. SEMINAR/COLLOQUIUM SERIES AND WORKING GROUPS

The Institute hosts seminar and colloquium series (whose frequency varies between twice a week to once a month) covering a wide spectrum of mathematical areas and related fields:

- (1) *Actuarial Science Financial Mathematics Seminar*
- (2) *Actuarial Science & Financial Mathematics Group Meetings*
- (3) *Algebraic Combinatorics Seminar*
- (4) *Arithmetic and Geometry Seminar*
- (5) *Colloquium/Seminar in Applied Mathematics*
- (6) *Fields Analysis Working Group*
- (7) *Fields Industrial Optimization Seminar*, co-sponsored by MITACS
- (8) *Fields Seminar Series on Quantitative Finance*, co-sponsored by MITACS
- (9) *Fields-Physics Colloquium*
- (10) *Geometry and Logic Seminar*
- (11) *Operator Algebra Seminar*
- (12) *PRMIA Risk Management Seminar*
- (13) *Set Theory Seminar*
- (14) *Toronto Probability Seminar*

## 8. CENTRE FOR MATHEMATICAL MEDICINE (CMM)

The Institute benefits from a unique position in proximity with one of the world's top centres for medical research. The collaboration between mathematicians, statisticians and information scientists plays an ever increasing role in the advancement of biological and medical research. The Centre for Mathematical Medicine, hosted by the Fields Institute and led by co-directors S. Sivaloganathan and J. Drake, serves as a nidus for researchers from medicine and mathematics, nurturing new interdisciplinary research ventures and initiatives. It receives support from Fields of \$100,000 per annum, through the Institute's increased funding from the Ontario Ministry of Training, Colleges and Universities (MTCU). The Centre organizes workshops, panel discussions, regular seminars, supports postdoctoral fellows, and fosters collaboration and networking with visiting scientists from around the world.

## 9. MULTI-DISCIPLINARY ACTIVITIES

The Institute supports research and scientific activities spanning the whole spectrum of mathematical sciences and related disciplines:

- (1) Health and Life Sciences
  - (a) *Computational Neuroscientists in Upper Canada (CNUCs) Workshop*, October 20, 2010
  - (b) *Centre for Mathematical Medicine Seminar Series* (monthly seminar attended also by clinicians from local research hospitals)
  - (c) *Variability Round Table*, September 28 – October 1, University of Ottawa
  - (d) *CMM Brain Neuromechanics Workshop*, July 26–28, 2010
- (2) Financial Mathematics
  - (a) *IFID Conference on Lifecycle Theory*, October 28, 2010
  - (b) *Quantitative Finance Seminar* (monthly)
  - (c) *Actuarial Science & Financial Mathematics Seminar* (monthly)
- (3) Industrial Collaboration
  - (a) *2010 Fields-MITACS Industrial Problem-Solving Workshop* (with projects from biology, medicine, engineering, finance), August 16–20, 2010
  - (b) *Fields Industrial Optimization Seminar* (monthly seminar)
- (4) Information Sciences
  - (a) *Toronto Quantum Information Seminar* (weekly seminar)
  - (b) *Research Meeting and School on Distributed Computing by Mobile Robots*, August 15–18, 2010, Carleton University
  - (c) *Fields–Perimeter Institute Workshop on Random Matrix Techniques in Quantum Information Theory*, July 4–6, 2010, at Perimeter
  - (d) *14th Developments of Language Theory*, August 17–20, 2010, University of Western Ontario
  - (e) *Selected Areas in Cryptography (SAC) Workshop*, August 12–13, 2010, University of Waterloo
- (5) Physics and Engineering (see also (4) above)
  - (a) *Workshop on Fluid Motion Driven by Immersed Structures*, August 9–13, 2010
  - (b) *Workshop on Hybrid Dynamic Systems*, July 29–31, 2010, University of Waterloo

## 10. EDUCATION

- (1) *JUMP (Junior Undiscovered Mathematics Prodigies)* is a charitable organization promoting a numeracy program which was initially developed in 1998, at Fields, by mathematician, author and award-winning playwright John Mighton (Order of Canada 2010). The JUMP Math Program is primarily intended to be used by teachers in the classroom, and offers materials intended to cover the curricula for grades 1 to 8. While JUMP Math has developed into a mainstream set of curriculum-based resources, it has kept its flexible nature and can be employed as a support in after-school tutoring by schools and community organizations, as well as by parents at home.
- (2) *Fields Mathematics Education Forum Meetings*

- (3) *Fields Math Circles*: weekly sessions, held at Fields, offering extracurricular high school-level mathematics activity and preparation for mathematics competitions
- (4) *Windows in Elementary Mathematics*: the goal of the project is to provide students, parents and teachers with insight into how mathematicians think through interviews with mathematicians
- (5) *Math Performance Festival*: for grades 4–6 students and teachers
- (6) *Caribou Mathematics Competition*: for grades 3–5 students, administered through Brock University
- (7) *Centre for Education in Mathematics and Computing (CEMC) Think about Math! Conference*, University of Waterloo: a meeting designed to ignite enthusiasm in high school girls for mathematics
- (8) *Big Ideas in Mathematics Symposium*

## 11. TARGETING NON-ACADEMIC SECTORS AND INDUSTRIAL COLLABORATION

Through various initiatives targeted outside the academic community, the Institute is able to maintain an extensive network of connections with mathematicians working in the private or government sector. A unique feature of the Fields Institute, in comparison with mathematical institutes world-wide, is its program of incubating mathematically oriented start-up companies, such as  $R^2$  *Financial Technologies Inc.* and *Sigma Analysis and Management*.

The following are Fields Corporate Affiliate Members:

- (1) Algorithmics
- (2) General Motors
- (3) QWeMA Group Inc.
- (4)  $R^2$  Financial Technologies Inc.
- (5) Sigma Analysis and Management

The *Commercial and Industrial Mathematics Program*, the start-up company incubation program, as well as the *Centre for Mathematical Medicine* all help building and maintaining strong links with the non-academic sector. The *Mathematics-in-Industry Case Studies (MICS)* journal is especially effective in this regard since it provides a venue for sharing ideas among academic and industrial researchers and a learning opportunity for newcomers and students.

The Institute participates in the MITACS Network, and in particular organizes industrial problem solving workshops, such as:

- (1) *Math-in-Medicine Study Group Biomedical Problems Solving Workshop*: co-sponsored by Fields, MITACS and the Oxford Centre for Collaborative Applied Mathematics (OCCAM), June 22–26, 2009 at Fields
- (2) *Fields–MITACS Industrial Problem-Solving Workshop*: August 16–20, 2010 at Fields
- (3) *Industrial Optimization Seminar*: held at Fields

## 12. PUBLIC EVENTS

The Institute has been very successful in attracting attention and increasing awareness of the public with regard to the mathematical sciences. The flexibility of the Institute's programming allows to take advantage of special opportunities to have the world's most prominent researchers deliver public lectures attended by hundreds.

Recent examples (all held at Fields) include:

- (1) *Fields Distinguished Lecture Series*:
  - (a) Cédric Villani (2010 Fields Medal), November 1, 3, 4, 2010
  - (b) Shing-Tung Yau (1982 Fields Medal and 2010 Wolf Prize), January 19–21, 2011
- (2) *Nathan and Beatrice Keyfitz Lectures*: Robert C. Merton (Nobel Prize 1997), April 15, 2010; George Lakoff, March 14, 2011
- (3) *The Man Who Saved Geometry*: a preview screening of a TVOntario documentary about Donald Coxeter (September 9, 2009)

- (4) *The Shape of Content*: an event to celebrate creative writing in mathematics and science (October 13, 2009)

### 13. NEW INITIATIVES

- Program structure:
  - (1) *1–2 month Focus or Thematic Programs* will be organized concurrently with major Thematic Programs at the Institute.
- International visibility and cooperation:
  - (2) *Fields Medal Symposium*: Starting with the Institute’s 20th Anniversary in 2012, this Symposium will take place at Fields once a year, and will feature an address or a series of lectures for a general scientific audience by one of the most recently announced Fields Medalists. This three-day event will also include lectures and panel discussion by other top international scientists on themes related to the work of the Medalist. A lecture of interest to the general public will be given either by the Fields Medalist or by another participant. The Symposium will be promoted in the Canadian and international press, and will be broadcast live throughout the world via the Institute’s interactive videoconferencing facilities.
  - (3) *Unité Mixte de Recherche (UMR)* of the French *Centre National de la Recherche Scientifique (CNRS)*: An agreement is being developed between Fields and the CNRS, establishing the Institute as an UMR of the latter.
  - (4) *Fields–Perimeter Africa Fellowship*: This Fellowship will be awarded to support a young researcher from Africa to participate in programs at the Fields Institute and/or the Perimeter Institute, for a duration of six or twelve months.
  - (5) The Fields Institute, the *Instituto Nacional de Matemática Pura e Aplicada (IMPA)* in Rio de Janeiro, and the *Weierstrass Institute for Applied Analysis and Stochastics (WIAS)* in Berlin, were invited to present bids in a competition to host a Stable Office of the *International Mathematical Union (IMU)*, at the IMU General Assembly meeting in Bangalore, India, in August 2010, in connection with the International Congress of Mathematicians 2010 in Hyderabad. The Fields Institute and the City of Toronto offered the IMU a unique opportunity, in terms of openness to people and ideas, diversity of the Institute’s scientific activities, and its global outreach (both to traditional centres of mathematics excellence and to the developing world). Neither Fields nor IMPA, however, were able to offer the same amount of government financial support as WIAS, whose bid was successful. It is this lack of governmental support that the Canadian mathematical community had already experienced in 2004 and 2008 with two Canadian bids to bring the International Congress of Mathematicians to Montreal – joint efforts by the learned societies, the Institutes and MITACS that were unfortunately unsuccessful.
- Unique training opportunities:
  - (6) *Fields–MITACS Undergraduate Summer Research Program*, held at Fields July – August 2010 (and planned for 2011): 20 undergraduate students spent eight weeks at Fields carrying out research in groups on various mathematics projects, ranging from infectious disease modelling to combinatorial game theory
  - (7) *Fields Undergraduate Network (FUN) Meetings*, held at Fields and its partner Universities: a series of workshops aimed at undergraduate students interested in research-level mathematics, including talks by researchers and a panel discussion
- Infrastructure:
  - (8) IT initiatives: The Institute has undertaken an ambitious video-conferencing project which includes live streaming of lectures, facilities for remote participants to interactively take part in the event, as well as automatic audio, video and slide capturing of a presentation for archival storage. Major improvements of the Institute’s web presence are also currently underway, in particular the setting up of blogs and wikis, which provide an important tool to reach the



general public, in particular pre-University students, as well as to facilitate communication between scientists.

- (9) Fields has recently become a Charter Member of the *Science Media Centre of Canada*.