Science

The current foundations of probability theory were set in 1933 by Kolmogorov, but despite their depth, beauty and development into ubiquitous concepts such as martingales, it is only in recent years that probability has moved to center stage in terms of Fields Medals and hiring by departments of mathematics. Fortunately probability is an area of remarkable and traditional strength in Canada. This is well recognized internationally: three Canadian probabilists have been elected to the Royal Society of London in the last 6 years, and 6 have been selected as ICM speakers (out of 29 from Canada) since 1986.

The two traditional foci in Canada were the Ottawa area, around D. Dawson and M. Csorgo, and UBC, with its initial strength originally built around J. Walsh and R. Chacon. The UBC group has continued to develop into an extremely strong school whose senior leaders are now M. Barlow, D. Brydges, E. Perkins and G. Slade. The focus in eastern Canada is now centered in the Toronto area where there is a very active young group at U. Toronto (I. Binder, K. Khanin, J. Quastel, J. Rosenthal, B. Virag) as well as strength at York U. (N. Madras and T. Salisbury). There is a strong group focused on the interface of probability and discrete mathematics at McGill (L. Devroye and B. Reed). These groups have exceptional junior researchers who will continue this strong tradition in Canadian mathematics into the next generation. Notable here are recent Sloan Fellows Omer Angel (UBC) and Balint Virag (U. Toronto). Other areas of strength in probability are Concordia U. (L. Popovic, W. Sun, and X. Zhou) U. Ottawa (G. Ivanov and D. McDonald) and U. Alberta (M. Kouritzin and B. Schmuland). The region of Montreal is well represented in the ever more important areas of actuarial science and finance with young researchers M. Bédard and M. Morales of UdeM, A. Roch and J.F. Renaud of UQAM, and C. Labbé of HEC Montréal.

Probability touches on a wide variety of subjects, merging into analysis and differential geometry at one end, and discrete mathematics at the other; important developments tie it quite closely to theoretical physics, in particular statistical physics. Probability is mathematics that is very visible outside mathematics, where applications include mathematical biology, computer science, and financial mathematics, where the impact of Itô's theoretical work on stochastic calculus led to his Gauss Prize at the 2006 ICM. Actuarial science and finance are attracting more and more graduate students, because of the interesting research problems and the employment opportunities in both academic and private sectors.

Areas of strength in Canada include stochastic PDE, statistical physics, Markov chain Monte Carlo, random media, interacting particle systems, population and genetic models and random matrices. The pioneering work of J. Walsh, D. Dawson and E. Perkins on stochastic PDE and measure valued processes has had a huge impact on the worldwide development of these fields. Don Dawson was the founder of what are now called Dawson-Watanabe superprocesses, and he and Perkins developed the fundamental properties of these processes which have become important tools in a variety of models in ecology and immunology as well as genetics. Recent highlights are Quastel's work on invariance of white noise for the Kortweg-deVries equation and effect of noise on traveling fronts, and Perkins' work on pathwise uniqueness for parabolic SPDE's with Hölder continuous coefficients. Quastel was an invited speaker at the 2010 ICM.

In statistical physics, Brydges and Slade are the mathematical leaders in applications of the renormalization group method and especially the lace expansion to stochastic models such as self-avoiding walk and branched polymers. In addition N. Madras and C. Soteros (U. Sask.) work on problems arising from the statistical mechanics of polymers such as entanglements. Brydges was an invited speaker at the 2010 ICM.

N. Madras and J. Rosenthal are leaders in obtaining rates of convergence for Markov chains to their equilibrium, motivated by Monte Carlo applications in statistics and statistical physics. M. Barlow is a leading probabilist in the world working in heat kernels for irregular and random media. L. Popovic and X. Zhou (Concordia) and B. Schmuland have done fundamental work on related models used in population genetics. Balint Virag is a major player in the relatively new and important fields of random matrices and determinantal processes. His work on stochastic Airy operators has attracted international attention, including a highly prestigious invitation to speak at Harvard's "Current Developments in Mathematics" on the work. In addition, his graduating student A. Bloemendal discovered a PDE characterization of the GUE Tracy-Widom distribution, and will now take the Simons Postdoc at Harvard, perhaps the most sought after postdoctoral position in mathematics.

Future Trends. Pure mathematicians love subjects with surprising connections to other parts of mathematics. The Schramm-Loewner Evolution (SLE) is a favourite from this point of view because it links beautiful results in the theory of complex variables with famous problems in statistical physics. There is a long way to go in this direction because conformal quantum field theory is a much larger collection of ideas than currently find expression in SLE. Random matrix theory also has enduring interest. It has captured the interest of the world's greatest analysts, for example, Terence Tao and H.T.Yau. Progress in this subject sheds light on the famous delocalisation transition in theoretical physics, and in a completely different direction, faculty and postdocs (Fotini Markopoulou, Razvan Gurau) at the Perimeter Institute study random geometry for insight into gravity and spacetime. Meanwhile down-to-earth problems are being imported into probability from analysis of risk in finance, computer science, discrete mathematics and biology, particularly genetics. It is possible that hard optimisation problems such as the travelling salesman problem are not intractable when the goal is weakened to finding acceptable approximations to most probable cases. In genetics the search for best models and properties of philogenetic trees will continue.

Research Funding

The current research infrastructure in Canada is good, though there is considerable worry among young people about the stability of funding and whether recent decisions accurately reflect scientific quality of proposals. The NSERC discovery grants enable individual researchers to use funds in the most appropriate way for their own projects, while the three Institutes are able to provide some funding for some larger projects. Particularly useful from the Institutes, and from BIRS, is support for small (about 40 people) focused workshops – these are much more valuable scientifically than larger meetings.

The main barriers we see are: 1. Low overall level of DGs, which mean that even a senior researcher with a large grant (compared to the distribution of grants in Mathematics and Statistics) will find it hard to support a postdoc. We would like NSERC to

continue to keep the bureaucratic overhead of grant application low, to not be ashamed of having a high success rate in mathematics – small grants play an important energising role in our community – and to not get carried away with new programs to force interdisciplinary trends. In probability our young people are flexible and very alert to the beauty of interconnections. They will certainly follow them if their ambition and time to think is not subdivided into infinitesmal pieces. 2. Timing of NSERC postdocs. Since these are announced well after the regular mathematics hiring season (Feb), the best candidates have already accepted postdoctoral positions elsewhere. This means that NSERC postdocal positions are not usually able to fill the gap of funding for postdocs mentioned above. 3. Recent funding decisions which appear very erratic and leave the impression that researchers may not be adequately funded even if their research is first class by any scientific standards. It is hard to overemphasize the problem here, as we may lose some of our strongest young people to other countries where decisions appear to be less arbitrary.

International.

We consider the best avenue for international development is to continue to provide money for DGs which will enable individual researchers to (a) make international visits, and (b) invite international visitors in a scientifically productive way. Taking money away from DGs for international projects would lead to a less efficient allocation of resources than at present.